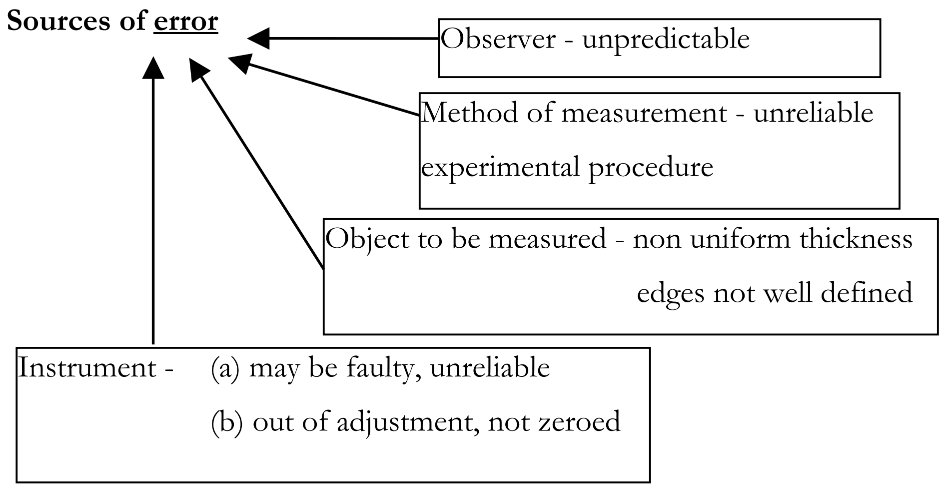
Error and Uncertainty

All that any experimental procedure can do is to give a value for the result that we can say may be near the **true value**. We can never say that we **know** the true result, only that we have a result that may lie within a **range of uncertainty**. For example, as a result of a number of measurements we may have a **best estimate** of the true value for the acceleration due to gravity, g, of 9.9 ms^{-2} and also be confident that our *uncertainty* is ± 0.1 ms^{-2}, i.e. g is between 9.8 and 10.0 ms^{-2}. If we are lucky then there may be an accepted value (e.g. g = 9.81 ms^{-2}) with which we may compare our result. If the accepted value lies within our range of uncertainty then we have performed a good experiment.

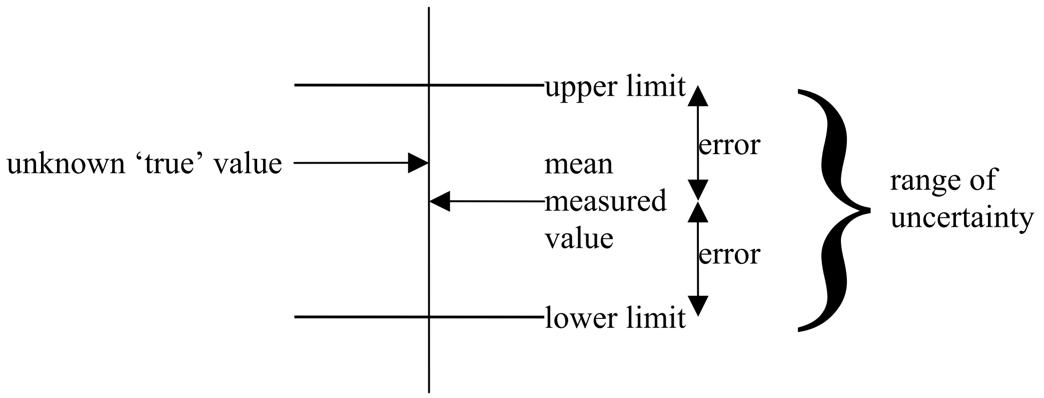


Types of error

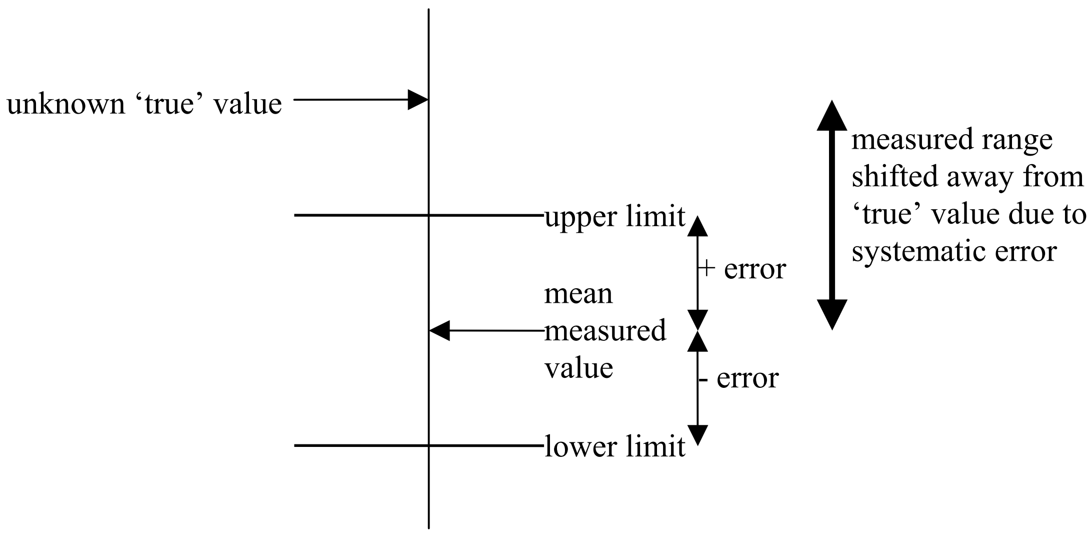
<u>Systematic error</u>

A systematic error is one associated with a particular measuring instrument or experimental technique, e.g. using a metre rule which has had the first 10 cm cut off, making all measurements 10 cm too high, or trying to find the acceleration due to gravity using an object that is subject to significant air friction, or in an experiment requiring a stable background temperature of 20ºC,

having the temperature be too high. Each of these will have the effect of shifting all results by a significant amount in the same direction, known as the systematic error.



the above system has no, or negligible, systematic error



The treatment of systematic errors is very difficult and requires experience. The experimental technique (or the offending measuring device) must be modified. For our purposes it is sufficient that we recognise the existence of a systematic error. If all other errors have been included in the **measured uncertainty range** and the accepted value still lies outwith this range then:

(a) we must say that there has been some systematic error present in the experimental procedure or in the equipment used, shifting all measurements too high or too low, and

(b) we must try to suggest where the systematic error may lie.

Random error

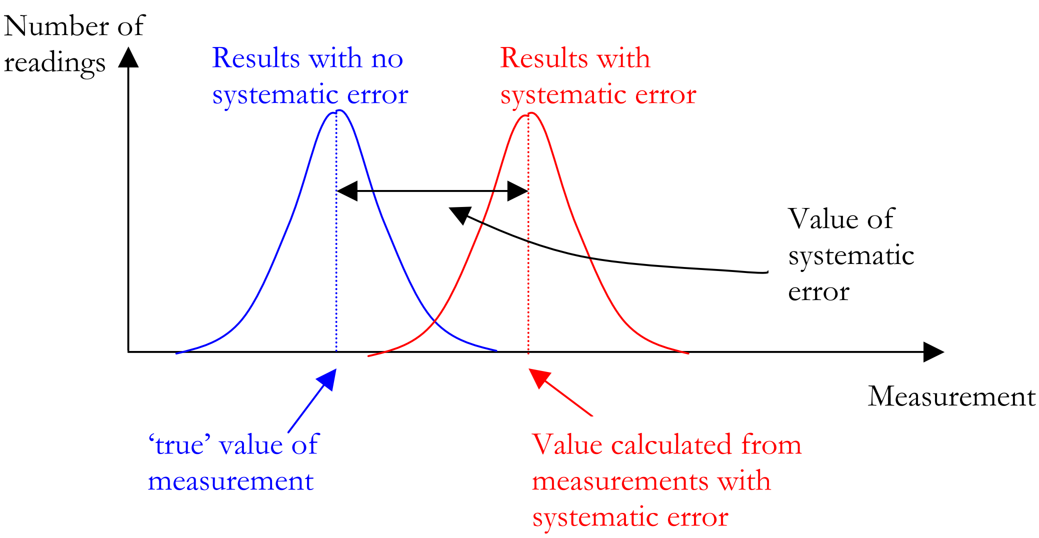
The random error in experimental results is due to lack of observer precision, perhaps in misreading an analogue scale due to parallax. This will result in a spread of results, even in the most carefully designed of experiments. Due to the random nature of these errors, there is an equal chance that they will be above or below the 'true' value. To mitigate against such errors, it is correct technique to take many readings and find the mean, even in the simplest of experiments. Because it is impossible to know the 'true' value, **the best estimate is the mean of repeat readings**.

The random error (also called the mean deviation) is then a measure of the spread of the repeat readings:

Random error, $\Delta_{ran} = \frac{R}{N}$ R = range (maximum - minimum)
N = number of repeat readings

Random error is reduced by increasing the number of readings, N. As N increases Δ_{ran} decreases.

The spread of experimental results about the mean value follows a normal distribution curve.



Reading error

The reading error in a measurement indicates **how accurately the scale can be read**. In analogue readouts the reading error is usually taken as plus or minus half the smallest division on the scale, but can be one fifth of the smallest division, depending on how accurately you think you can read the scale. In the end, it is down to your judgement.

In digital readouts the reading error is taken as plus or minus one digit on the last readout number. For example, if a digital ammeter readout was 42.3 mA, the reading error would be ± 0.1 mA.

Calibration error

The calibration error in a measurement **indicates how well the measuring instrument has been made**, and is usually quoted by the manufacturer. It is usually quoted as a percentage of the reading or a percentage of full scale deflection (f.s.d.).

e.g

instrument	*calibration error*
wooden metre stick	± 0.5 mm
steel rule	± 0.1 mm
mercury thermometer	± 0.5 °C
analogue electrical meter	2 % of f.s.d.
Digital electrical meter	0.5 % of f.s.d. + 1 digit

Total error

Where more than one physical quantity has been measured and the results combined to give a final result, e.g. c = $\frac{E_h}{m\,\Delta T}$, the

largest percentage error is taken to be the best estimate of the total error.

The total error can be expressed in many ways:

e.g.	length measured as 20.00 ± 0.05 m	time measured as t ± Δt
absolute error	0.05 m	Δt
relative error	$\frac{0.05}{20.00} = \frac{1}{400}$	$\frac{\Delta t}{t}$
percentage error	$\frac{0.05}{20.00}$ x 100% = 0.25%	$\frac{\Delta t}{t}$ x 100%

<u>Significant figures in error calculations</u>

When calculating an error, or a percentage error, work to one significant figure only unless the first figure is a '1', then quote error to two significant figures to avoid a rounding error.

example 1. Speed, v = 34.2 ± 4.3 ms^{-1}.

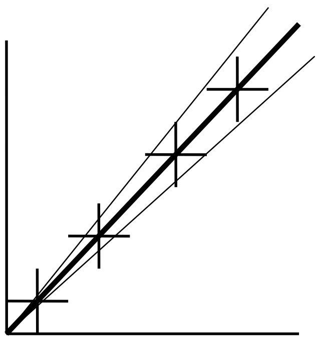
Round to v = 34 ± 4 ms^{-1}.

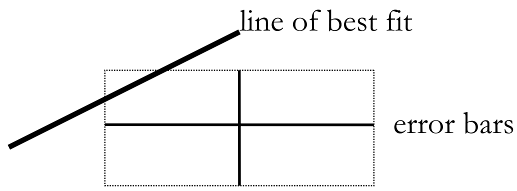
example 2. Gravitational field strength, g = 10.4 ± 1.8 Nkg^{-1}.

Round to g = 10 ± 1.8 Nkg^{-1}.

In both examples above the final significant figure in the result has been made meaningless because of the size of the error. **The rule here is to quote the result only to the first figure in the error.**

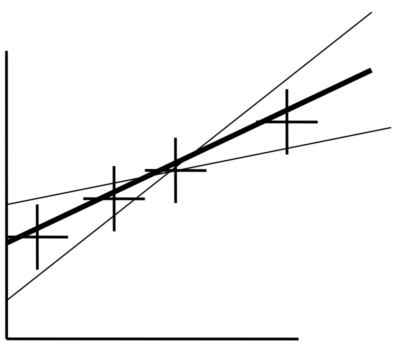
Error bars on graphs





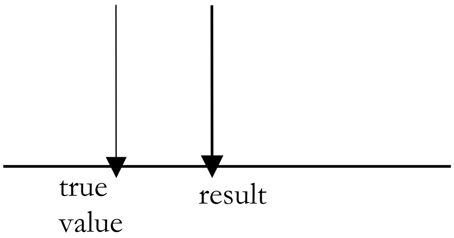
The length of the error bar indicates the magnitude of the error. Any line on a graph to show best fit should pass through the box defined by the error bars.

When the line passes through the origin, the error in the gradient is given by the range of the gradient as shown.

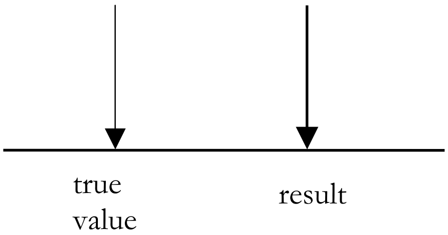


If the line is not to pass through the origin then the error in the gradient is the range in the gradients for best-fit-line to 'worst-fit-line'.

Terminology: "accurate" and "precise"



An accurate result is one that involves a small systematic error, i.e. it is near the 'true' value.



A precise result has a small random error, i.e. a small spread of results. However, it may not be accurate, i.e. it may have a large systematic error and so can be removed from the 'true' value.

Powers of ten and SI prefixes

Name	**Symbol**	**Value**
tera	T	x 10^{12}
giga	G	x 10^{9}
mega	M	x 10^{6}
kilo	k	x 10^{3}
centi	c	x 10^{-2}
milli	m	x 10^{-3}
micro	μ	x 10^{-6}
nano	N	x 10^{-9}
pico	p	x 10^{-12}